V Money, Inflation and Monetary Policy

1 Introduction

- Why do people hold money?
- Does money affect real decisions?
- Why does money affect real decision?
- What are the causes of inflation?
- What are the costs of inflation?
- How should monetary policy be conducted?

2 The OLG Model with Money (Samuelson, 1958)

In this model, time is discrete and individuals live for two periods (young and old). N_t individuals are born at time t and population grows at a constant rate n. Normalizing $N_0 = 1$ gives $N_t = (1+n)^t$.

- Utility function: $u(c_t, c_{2t+1})$
- Endowment: 1 unit of consumption good when young and no endowment when old
- Storage technology: One unit saved at time t yields $1+r$ units at time $t+1$

2.1 The Barter Equilibrium

Suppose the good received by the young is perishable , i.e. $r = -1$. Individuals would like to spread consumption over the two periods, but no trade can take place because of the absence of a double coincidence of wants (Figures 1 and 2).

- Decentralized equilibrium: All individuals consume all of their endowment when young and consume nothing when old.
- Decentralized equilibrium is not Pareto optimal!

2.2 The Introduction of Money

Suppose that at time zero the government gives to the old H completely divisible pieces of paper money and that the old and every generation thereafter *believe* that they will be able to exchange money for goods at price P_t in period t.

Then an individual born at time t chooses money demand M_t^d to maximize $u(c_{1t}, c_{2t+1})$ subject to

$$P_t(1 - c_{1t}) = M_t^d \quad \text{and} \quad P_{t+1}c_{2t+1} = M_t^d. \tag{1}$$

The first-order condition is

$$\frac{-u_1(c_{1t}, c_{2t+1})}{P_t} + \frac{u_{2t+1}(c_{1t}, c_{2t+1})}{P_{t+1}} = 0, \tag{2}$$

which gives the a money demand function

$$\frac{M_t^d}{P_t} = L\left(\frac{P_t}{P_{t+1}}\right). \tag{3}$$

The rate of return on money is given by $1 + g_t = P_t/P_{t+1}$, where g_t is the rate of deflation.

2.3 Equilibrium

Equilibrium in the money market implies:

$$(1+n)^t M_t^d = H. \tag{4}$$

Combining (3) and (4) gives

$$\frac{1+n}{1+g_t} = \frac{L(1+g_t)}{L(1+g_{t+1})}.$$

In steady state, $g = n$, that is, the rate of deflation must be equal to the growth rate of population.

2.4 Results

We have the following set of results:

- Money can have positive value.
- If money is valued, then introducing money allows for new trades.
- Assuming the economy reaches steady state, the introduction of money can lead to a Pareto optimal equilibrium.

2.5 Money in an Economy with Storage

Suppose that the rate of return from storage is $r > -1$.

- Case 1: If $r < n$, i.e., storage is not productive enough, then the barter equilibrium is still not Pareto optimal, the monetary equilibrium is Pareto optimal and storage will not be used in the monetary equilibrium (Figure 3).

- Case 2: If $r \geq n$, i.e., storage is sufficiently productive, then the barter equilibrium is Pareto optimal and there cannot be a monetary equilibrium.

Conclusion: If the barter equilibrium is not a Pareto optimum, there exists a monetary equilibrium that leads to a Pareto optimal; if the barter equilibrium is already a Pareto optimum, there cannot be a monetary equilibrium.

2.6 The Effects of Money Growth

Suppose that the nominal money stock grows at a constant rate σ. Then in steady state, inflation rate is $\sigma - n$. The effects of money growth on allocation of resources depend on how the new money is introduced into the economy. S

- Case 1: Suppose that the new money is introduced through lump-sum transfers to the old and that goods are perishable. Then the budget constraint (1) becomes

$$P_t(1 - c_{1t}) = M_t^d \quad \text{and} \quad P_{t+1}c_{2t+1} = \Delta M_t + M_t^d. \tag{5}$$

The solution is given by

$$\frac{M_t^d}{P_t} = L\left(\frac{P_t}{P_{t+1}}, \frac{\Delta M_t}{P_{t+1}}\right). \tag{6}$$

Suppose the new money is distributed equally among the old, then $\Delta H_t = N_t \Delta M_t$. The money market equilibrium condition remains the same as before, i.e., $N_t M_t^d = H_t = (1+\sigma)H_{t-1}$. In steady state,

$$1 + g = \frac{1+n}{1+\sigma} \quad \Rightarrow \quad g = n - \sigma \ \text{ (if } \sigma \text{ and } n \text{ are small).} \tag{7}$$

Result 1: Money is not superneutral because money growth affect the rate of return on money and thus affects the allocation of resources (Figure 4).

Result 2: The monetary equilibrium is not Pareto optimal.

If goods can be stored, then whether a monetary equilibrium exists depends on the rate of return on money. A necessary condition is

$$1 + g = \frac{1+n}{1+\sigma} \geq 1 + r,$$

that is, money growth should be sufficiently small.

- Case 2: Suppose the new money is introduced through interest payments to money holders. Then Then the budget constraint (1) becomes

$$P_t(1 - c_{1t}) = M_t^d \quad \text{and} \quad P_{t+1}c_{2t+1} = (1+\sigma)M_t^d. \tag{8}$$

In this case, the inflation is the same as in Case 1. However, money growth does not affect the rate of return on money because the new money in the form of interest payments to money holders compensate them for the additional inflation.

Result: Money is superneutral.

2.7 Summary

The above results can be summarized by the following two propositions:

Proposition 1: There can be a monetary equilibrium only if the barter equilibrium is not a Pareto optimum.

Proposition 2: Although monetary equilibria are not all Pareto optima, there is a Pareto optimal monetary equilibrium.

Shortcomings of this model: Money is valued only when it is not dominated in rate of return by any other asset. If storage is sufficiently productive, yielding a higher return than money, money is no longer valued. Monetary equilibria do not exist if rate of inflation is too high. However, in reality, money is dominated in rate of return by many assets, but it continues to be used even during hyperinflation periods.

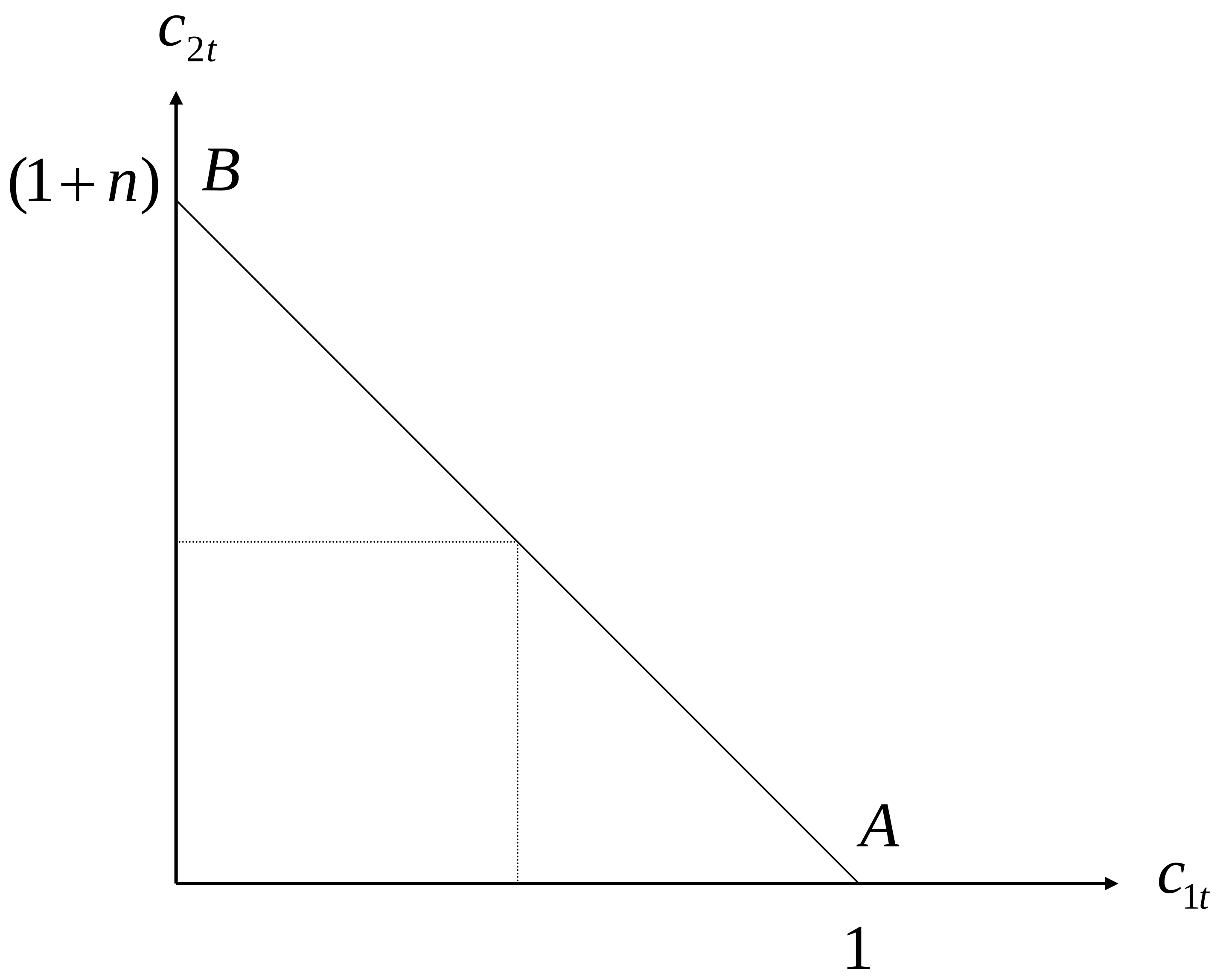

Figure 1: Society's Consumption Possibilities in Period t

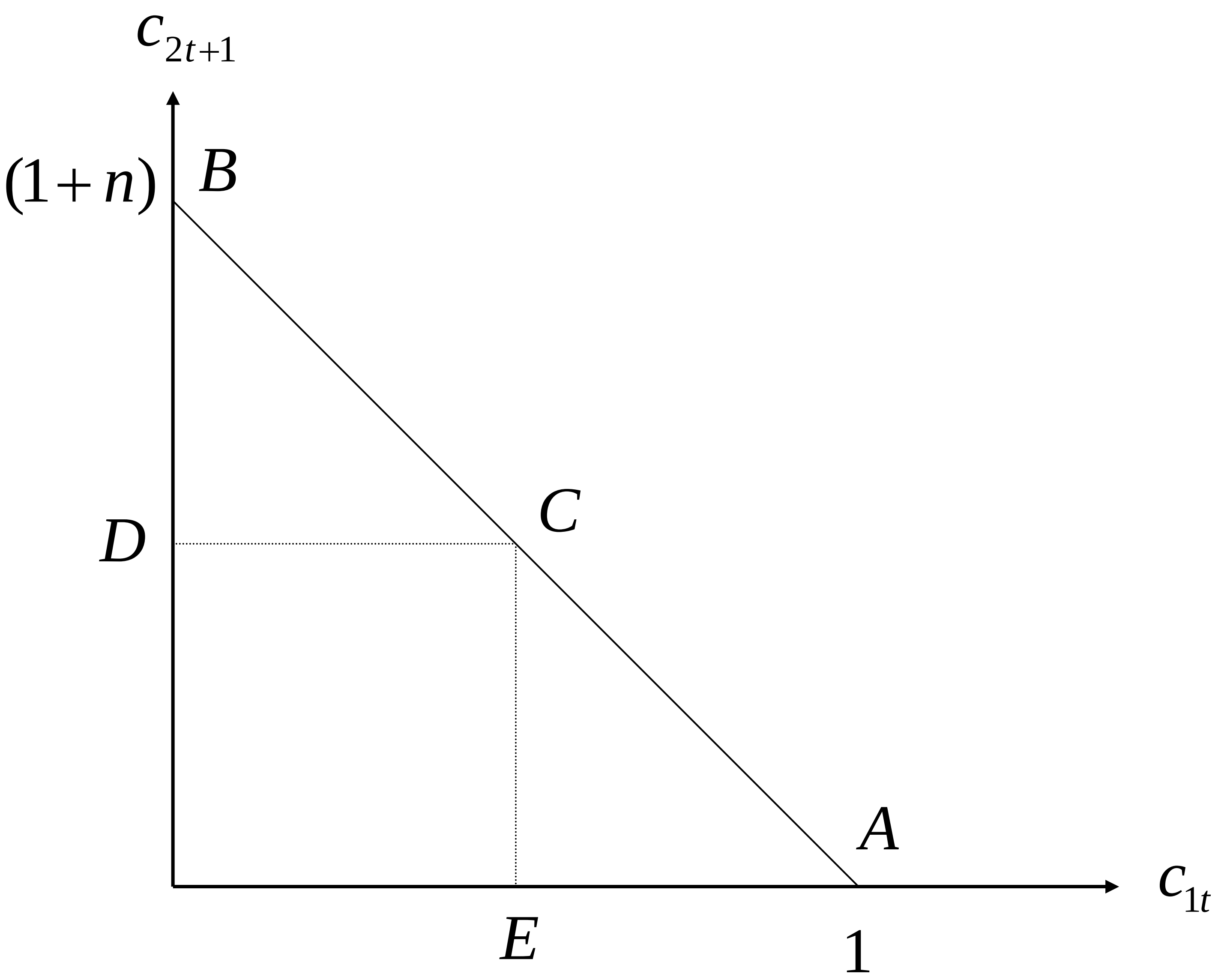

Figure 2: An Individual's Lifetime Consumption Possibilities

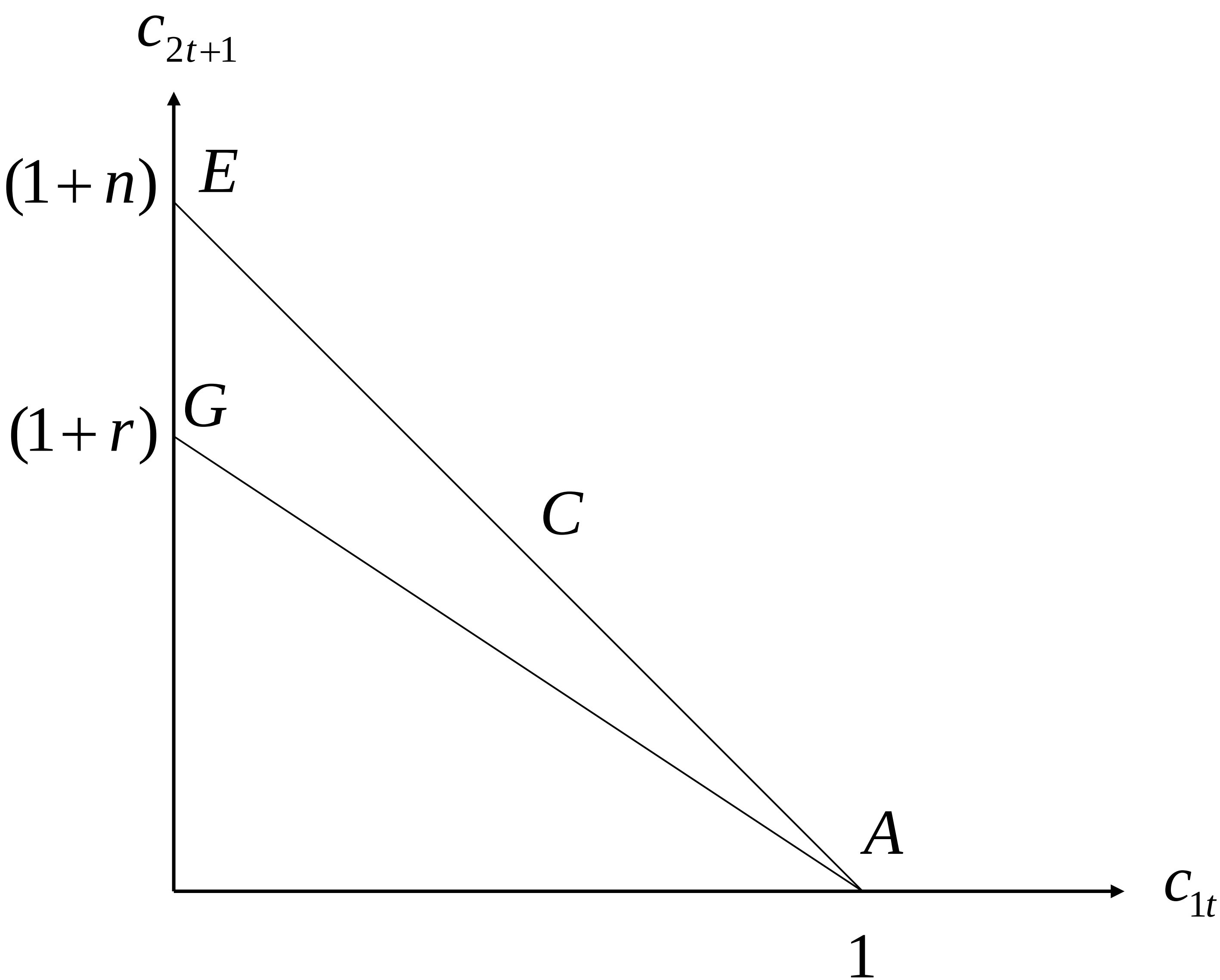

Figure 3: Equilibrium with Storage

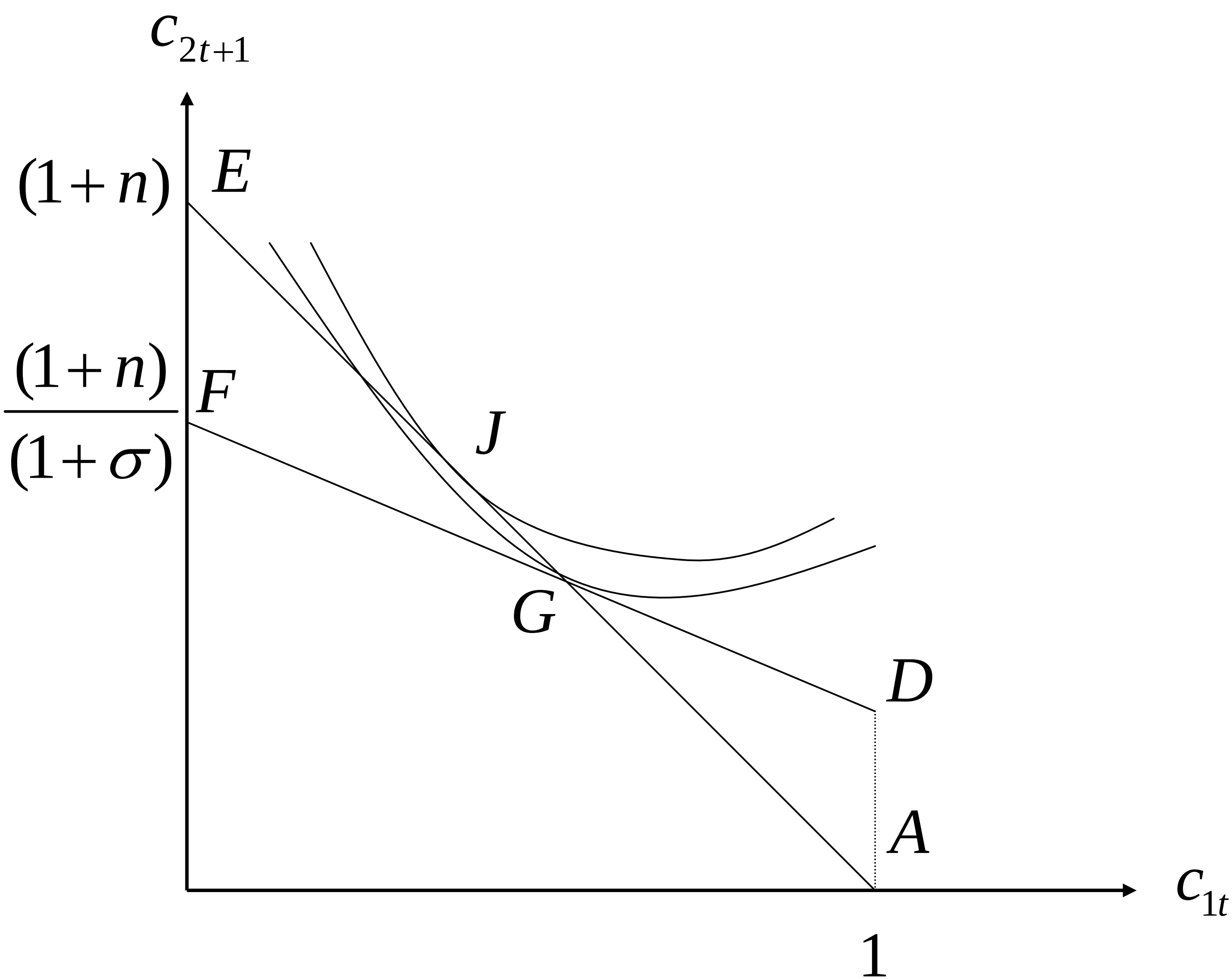

Figure 4: Welfare Effects of Increased Money Growth