Advanced Macroeconomics II

Assignment 2

(Submission Time: 3:30 pm, 15 June 2007)

1. In the AK model discussed in class, assume that the representative household's utility function is given by:

$$u(c) = \frac{c^{1-\theta} - 1}{1-\theta}, \quad \theta > 0. \tag{1}$$

Firm i's production function takes the following form:

$$y_i = Ak_i^{1-\alpha}x^{\alpha}, \quad A > 0, \quad 0 < \alpha < 1, \tag{2}$$

where y_i is output per worker, k_i is capital per worker and x is public expenditure per worker. Public expenditure is financed by a proportional tax, τ, on income. Assume that the government must always have a balanced budget.

(a) Find the dynamic equation or consumption in a competitive economy. What does it depend on?

(b) Show that, in equilibrium, output is given by an AK production function. [That is, output can be expressed as being proportional to the stock of capital.]

(c) How can the government maximize growth in a competitive economy? What happens when there are no taxes? What happens when $\tau = 1$?

(d) Is the competitive equilibrium socially optimal? Why?

2. In the Lucas (1988) model discussed in class, assume that the representative household's utility function is given by:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \tag{3}$$

where $\sigma > 0$. The production function is:

$$Y = AK^{\beta}(Nhu)^{1-\beta}h_a^{\gamma}, \tag{4}$$

where $0 < \beta < 1$, $\gamma \geq$, $0 \leq u \leq 1$ and $A > 0$ and $N > 0$ are constant. Human capital is accumulated according to

$$\dot{h} = \delta(hu)^{\alpha}[h(1-u)]^{1-\alpha}, \tag{5}$$

where $\delta > 0$ and $0 \leq \alpha \leq 1$.

(a) Find the first-order conditions for the representative household's optimization problems.

(b) Use the first-order conditions in (a) to obtain the growth rate of consumption as a function of h and k.

(c) How does the growth rate of human capital relate to the growth rate of physical capital?

(d) Find the growth rate of per capita output and the fraction of time spent in production.

(e) What is the optimal level of u? Is it the same as in (d)?

(f) In the decentralized economy, suppose the government imposes a proportional tax τ on all income. What is the steady-state growth rate of per capita output? How does the tax affect the growth rate? Explain your answer.

3. Consider the Romer (1990) model discussed in class. Suppose that the government subsidizes the purchases of intermediate goods (at a constant rate s) and finances the subsidy by a lump-sum tax T.

(a) How does the subsidy affect the quantities of intermediate goods? Explain why the quantities of intermediate goods respond to the subsidy.

(b) Find the steady-state equilibrium growth rate of output. How does the subsidy affect this growth rate? Provide the economic intuitions.

(c) What is the subsidy rate that can induce the decentralized economy to attain the social optimum?